JHEP HYPER VERSION

Received: *January 22, 2001*, Accepted: *February 12, 2001*

Comments on the thermodynamics of little string theory

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Abstract: We study the high energy thermodynamics of little string theory, using its holographic description. This leads to the entropy-energy relation $S = \beta_H E + \alpha \log E + O(1/E)$. We compute α and show that it is negative; as a consequence, the high energy thermodynamics is unstable. We exhibit a mode localized near the horizon of the black brane, which has winding number one around euclidean time and a mass that vanishes at large E (or $\beta \to \beta_H$). We argue that the high temperature phase of the theory involves condensation of this mode.

Keywords: p-branes, Black Holes in String Theory, AdS-CFT Correspondance.

JHEP02(2001)021

Contents

1. Introduction

Little String Theory (LST) [1] is a non-local theory without gravity which appears in two related contexts. One involves the study of vacua of string theory which contain Neveu-Schwarz (NS) fivebranes in the decoupling limit $g_s \to 0$. In this limit all string modes that live in the bulk of spacetime decouple, but the physics on the fivebranes remains non-trivial [2]. This gives rise to an interacting theory in $5+1$ or less dimensions with sixteen or fewer supercharges.

An alternative definition of LST involves string theory on Calabi-Yau (CY) spaces, at points in the moduli space of vacua where the CY space develops an isolated singularity [3]. Sending $g_s \to 0$ one again finds trivial physics in the bulk of the CY manifold, with interacting physics at the singular point.

In this paper we focus on the maximally supersymmetric vacuum of LST in $5+1$ dimensions, corresponding to N flat parallel $NS5$-branes with worldvolume $\mathbb{R}^{5,1}$. Most of the analysis generalizes trivially to the large class of vacua of LST constructed in [3], but we will not discuss the details here.

Holography relates LST to string theory in the near-horizon geometry of the fivebranes [4]. An important source of difficulty in studying detailed properties of the theory using this approach is the fact that the near-horizon geometry includes a "linear dilaton" direction, the real line $\mathbb{R}_\phi$ (which we will label by ϕ) along which the dilaton Φ varies linearly,

$$\Phi = -\frac{Q}{2}\phi\,, \tag{1.1}$$

with Q a model-dependent constant. The behavior (1.1) implies that the string coupling $\exp(\Phi) = \exp(-Q\phi/2)$ vanishes as $\phi \to \infty$ (the region far from the fivebranes), but it diverges as one approaches the fivebranes. Thus, the study of LST using holography involves solving the dual theory at strong coupling.

There are some situations in which the strong coupling problem mentioned above can be avoided. For example, a large class of observables in the theory can be identified by analyzing non-normalizable vertex operators whose wave-functions are exponentially supported in the weak coupling region $\phi \to \infty$ [4]. Also, one can study LST along its moduli space of vacua [5], since in such situations the strong coupling singularity associated with (1.1) is eliminated and one can study the theory in a weak coupling expansion.

In this paper we discuss another situation where LST becomes weakly coupled and thus amenable to a perturbative holographic analysis — the high energy density regime. As we review in section 2, LST has a Hagedorn density of states at very high energies,

$$\rho(E) = e^{S(E)} \sim e^{\beta_H E} . \tag{1.2}$$

The entropy is linear in the energy, and the inverse temperature β is constant,

$$\beta = \frac{\partial S}{\partial E} = \beta_H . \tag{1.3}$$

The thermodynamics is thus degenerate for very large energy, and it is of interest to compute finite energy corrections to (1.2), (1.3). We do this in section 3 and find that, as suggested in [6, 7], the density of states has the form

$$\rho(E) \sim E^{\alpha} e^{\beta_H E} \left[1 + O\left(\frac{1}{E}\right)\right] . \tag{1.4}$$

One of our main purposes is to compute the constant α. We find that α is negative and therefore the high energy thermodynamics is unstable. The density of states (1.4) gives rise to the temperature-energy relation

$$\beta = \frac{\partial \log \rho}{\partial E} = \beta_H + \frac{\alpha}{E} + O\left(\frac{1}{E^2}\right) . \tag{1.5}$$

Since α is negative, the temperature is above the Hagedorn temperature $T_H = 1/\beta_H$, and the specific heat is negative — increasing the energy of the system leads to a decrease of the temperature. This behavior is reminiscent of black holes in flat spacetime, which also have negative specific heat. In section 4 we argue that LST in fact undergoes a phase transition at or around the Hagedorn temperature T_H. In a euclidean time representation of the thermodynamics, a mode with winding number one around euclidean time goes to zero mass at T_H. It is likely that it becomes tachyonic above the Hagedorn temperature and condenses.

2. Classical NS fivebrane thermodynamics

The supergravity solution for N coincident near-extremal $NS5$-branes in the string frame is [8]:

$$ds^2 = -\left(1 - \frac{r_0^2}{r^2}\right) dt^2 + \left(1 + \frac{N\alpha'}{r^2}\right)\left(\frac{dr^2}{1 - r_0^2/r^2} + r^2 d\Omega_3^2\right) + dy_5^2 , \tag{2.1}$$

$$e^{2\Phi} = g_s^2 \left(1 + \frac{N\alpha'}{r^2}\right) . \tag{2.2}$$

$r = r_0$ is the location of the horizon, dy_5^2 denotes the flat metric along the fivebranes, and $d\Omega_3^2$ is the metric on a unit three-sphere. The solution also involves a non-zero NS $B_{\mu\nu}$ field which we suppress. The configuration (2.1), (2.2) has energy per unit volume

$$\frac{E}{V_5} = \frac{1}{(2\pi)^5 \alpha'^3} \left(\frac{N}{g_s^2} + \mu \right) , \tag{2.3}$$

where

$$\mu = \frac{r_0^2}{g_s^2 \alpha'} . \tag{2.4}$$

The first term in (2.3) is the tension of extremal $NS5$-branes and can be ignored for the thermodynamic considerations below (it is a ground state energy). μ measures the energy density above extremality and g_s is the asymptotic string coupling, which goes to zero in the decoupling limit.

The near-horizon geometry is obtained by sending $r_0, g_s \to 0$ keeping the energy density μ fixed. Changing coordinates to $r = r_0 \cosh\sigma$ and Wick rotating $t \to it$ to study the thermodynamics, one finds

$$ds^2 = \tanh^2 \sigma dt^2 + N\alpha' d\sigma^2 + N\alpha' d\Omega_3^2 + dy_5^2 , \tag{2.5}$$

$$e^{2\Phi} = \frac{N}{\mu \cosh^2\sigma} . \tag{2.6}$$

String propagation in this geometry corresponds to an "exact conformal field theory",

$$H_3^+ / \, \mathrm{U}(1) \times \mathrm{SU}(2)_N \times \mathbb{R}^5 , \tag{2.7}$$

where

$$H_3^+ = \frac{\mathrm{SL}(2, \mathbb{C})_N}{\mathrm{SU}(2)_N} \tag{2.8}$$

is the euclidean AdS_3 CFT which plays an important role in the AdS-CFT correspondence (see e.g. [9, 10]); the coset $H_3^+ / \, \mathrm{U}(1)$, parametrized by (σ, t) in (2.5), is a semi-infinite cigar [11]. The second factor in (2.7) describes the angular three-sphere.[1] The radius of the three-sphere can be read off (2.5),

$$R_{\text{sphere}} = \sqrt{N\alpha'} . \tag{2.9}$$

Finally, the third factor in (2.7) describes the spatial directions along the fivebranes.

The number of fivebranes N determines the levels of the SL(2) and SU(2) current algebras in (2.7), (2.8). More precisely, since (2.7) is a background for the superstring, the worldsheet theory contains fermions; the total level N of the SU(2) and SL(2)

[1] As is well known, CFT on a three-sphere with a suitable NS $B_{\mu\nu}$ field is described by the SU(2) WZW model.

current algebras receives a contribution of $N-2$ and $N+2$ respectively from the bosons, and $+2$ and -2 respectively from the fermions. The total central charge is

$$\frac{3(N-2)}{N}+\frac{3(N+2)}{N}+5+8\cdot\frac{1}{2}=15\,, \tag{2.10}$$

which is the correct value for the superstring. Note also that the background (2.7) is exact so we do not have to worry about α' (or, equivalently, $1/N$) corrections to the geometry.

The absence of a conical singularity at the tip ($\sigma = 0$ in (2.5)) requires the circumference of the cigar to be

$$\beta_H = 2\pi\sqrt{N\alpha'}\,. \tag{2.11}$$

Thus, euclidean time lives on a circle of radius $\sqrt{N\alpha'}$, and the temperature of the system is $T_H = 1/\beta_H$. In particular, the temperature is independent of the energy density μ, which determines the value of the string coupling at the tip of the cigar (2.6).

The fact that the temperature is independent of the energy means that the entropy is proportional to the energy (see (1.3)). Therefore, the free energy is expected to vanish,[2]

$$-\beta\mathcal{F} = S-\beta E = 0\,. \tag{2.12}$$

In general in string theory the free energy is related to the string partition sum via

$$-\beta\mathcal{F} \equiv \log Z(\beta) = Z_{\rm string}\,, \tag{2.13}$$

where $Z_{\rm string}$ is the single string partition sum, given by a sum over connected Riemann surfaces [13]. The string path integral should be performed over geometries in which euclidean time is compactified on a circle of radius $R = \beta/2\pi$ (asymptotically). For high energies one expects the thermodynamics to be dominated by the black brane geometry (2.1), (2.5) and thus the free energy is proportional to the partition sum of string theory in the background (2.7).

The string partition sum $Z_{\rm string}$ can be expanded as follows:

$$Z_{\rm string} = e^{-2\Phi_0}Z_0 + Z_1 + e^{2\Phi_0}Z_2+\cdots\,, \tag{2.14}$$

where $\exp(\Phi_0)$ is the effective string coupling in the geometry (2.5) and Z_h the genus h partition sum in the background (2.7). Although the string coupling varies along the cigar (see (2.6)), it is bounded from above by its value at the tip,

$$e^{2\Phi_0} = \frac{N}{\mu}\,. \tag{2.15}$$

[2]See [12] for a related discussion in the low energy gravity approximation.

Therefore, it is natural to associate (2.15) with the effective coupling in (2.14). We see that the string coupling expansion in the background (2.7) provides an asymptotic expansion of the free energy in powers of $1/\mu$.

The leading term in the free energy (2.13), (2.14) goes like

$$-\beta\mathcal{F} = \frac{\mu}{N} Z_0 \tag{2.16}$$

and corresponds to a free energy that goes like the energy (Z_0 is proportional to the volume of the fivebrane). This term is expected to vanish (see (2.12)), and therefore we conclude that the spherical partition sum in the background (2.7) should vanish. The fact that this is indeed the case follows from the results of [14], who analyzed a closely related problem, the partition sum of vacua including $N = 2$ Liouville as a factor. $N = 2$ Liouville is relevant for our problem since, as argued in [5], it is equivalent to the euclidean cigar SCFT. Also, the reasoning of [14] can be applied directly to the cigar. We will next briefly review the argument of [14] for the vanishing of Z_0, first for the cigar and then for $N = 2$ Liouville.

Normally in string theory the partition sum on the sphere is said to vanish, due to the volume of the Conformal Killing Group (CKG) of the sphere, $\mathrm{SL}(2,\mathbb{C})$. If the target space is non-compact, the partition sum is actually proportional to $V/\mathrm{vol}(\mathrm{SL}(2,\mathbb{C}))$ where V is the divergent volume of spacetime. Thus, at first sight the partition sum is ∞/∞, i.e. ill defined. However, in most situations one is actually interested in the partition sum per unit volume. E.g. if the vacuum is translationally invariant in the non-compact directions, the partition sum per unit volume is the lagrangian density in this vacuum (i.e. the classical cosmological constant), and it vanishes due to the volume of the CKG.

In the euclidean cigar background $H_3^+/\,\mathrm{U}(1)$, the above discussion has to be reexamined. There is no longer a reason to divide by the volume of the cigar: the background is not translationally invariant in ϕ, and in any case, the holographic prescription of [4] relates the free energy to the full string partition sum and not to the partition sum per unit volume (see (2.13)).

The ratio of the volume of the cigar to the volume of the CKG is finite in this case. The volume of H_3^+ contains precisely the same kind of divergence as that of the CKG. Since the volumes of the SU(2) and U(1) in (2.7), (2.8) are finite, we conclude that the partition sum of string theory in the cigar background is non-zero.

This conclusion is indeed correct in the bosonic string; in the superstring one has to take into account the fermionic zero modes. The CKG of the sphere is generalized to a Superconformal Killing Group (SCKG), but the added zero modes are cancelled by fermionic zero modes of the $N = 1$ SCFT on the cigar. Thus, it appears that in the superstring as well the partition sum in the background (2.7) is finite, in contradiction to (2.12).

This conclusion is incorrect because of an interesting property of the $N = 1$ SCFT on the cigar $H_3^+/\,\mathrm{U}(1)$. It turns out that this model has an "accidental" global $N=2$ superconformal symmetry. In fact it is a special case of the Kazama-Suzuki construction [15]. Thus, the SCFT has twice as many fermionic zero modes as one would naively guess, and integrating over them leads to the vanishing of the spherical partition sum [14].

The above argument was made in the language of the SCFT on the cigar. In the dual [5] $N=2$ Liouville theory, the vanishing of the spherical partition sum can be alternatively exhibited as follows. The $N=2$ Liouville lagrangian is

$$\mathcal{L}=\mathcal{L}_0+\lambda\int d^2\theta e^{-2/(\alpha' Q)(\phi+ix)}+\bar\lambda\int d^2\bar\theta e^{-2/(\alpha' Q)(\phi-ix)}\,. \tag{2.17}$$

$\mathcal{L}_0$ is the free field lagrangian for ϕ, x in a linear dilaton background (1.1). The linear dilaton slope Q can be determined by comparing the central charge of (2.17), $c=3+(3\alpha' Q^2/2)$ to that of the cigar CFT (2.8), $c=3+(6/N)$. This leads to

$$Q=2/\sqrt{N\alpha'}\,. \tag{2.18}$$

ϕ is a rescaled version of σ; far from the tip of the cigar one has $\phi\simeq\sqrt{N\alpha'}\sigma$. x can be thought of as the T-dual of θ; it lives on a circle of radius $\sqrt{\alpha'/N}$. λ is the $N=2$ Liouville coupling. $\int d^2\theta$ is an integral over half of the worldsheet superspace, $\int d^2\theta = G^+_{-1/2}\bar G^+_{-1/2}$. The operator $e^{-2/(\alpha' Q)(\phi+ix)}$ is chiral, i.e. it is annihilated by G^-, $\bar G^-$.

A standard scaling analysis shows that the partition sum on the sphere goes like

$$Z_0\sim(\lambda\bar\lambda)^{1/N}\,. \tag{2.19}$$

Thus, to compute Z_0 we can differentiate a number of times w.r.t. λ, $\bar\lambda$. Consider e.g. $\partial^2_\lambda\partial_{\bar\lambda}Z$. This is a three point function with two insertions of $\int d^2z\int d^2\theta\exp\left[-(2/\alpha' Q)(\phi+ix)\right]$ and one $\int d^2z\int d^2\bar\theta\exp\left[-(2/\alpha' Q)(\phi-ix)\right]$. The SCKG allows us to fix the locations of the three operators on the sphere (i.e. drop the z integrals), and also drop two of the three θ integrals. One possible gauge fixing is

$$\partial^2_\lambda\partial_{\bar\lambda}Z=-\langle 0|e^{-2/(\alpha' Q)(\phi+ix)(z_1)}e^{-2/(\alpha' Q)(\phi+ix)(z_2)}G^-_{-1/2}\bar G^-_{-1/2}e^{-2/(\alpha' Q)(\phi-ix)(z_3)}|0\rangle\,. \tag{2.20}$$

Using the fact that $G^-_{-1/2}$, $\bar G^-_{-1/2}$ commute with $\exp(-2/(\alpha' Q)(\phi+ix))$, we can push them to the left, until they annihilate the vacuum $\langle 0|$. Thus the amplitude vanishes.

Note that the vanishing of Z_0 relies on $N=2$ worldsheet supersymmetry, and worldsheet conformal invariance, but not on spacetime supersymmetry. In fact, the non-extremal vacua we are discussing break all spacetime supersymmetry.

We see that to leading order in $1/\mu$ the free energy vanishes, in agreement with the energy-entropy relation (1.2) implied by the classical cigar geometry. To compute $1/\mu$ corrections, we have to examine string loop effects in the background (2.7). In the next section we will study the one loop correction Z_1 (see (2.14)).

3. The leading $1/\mu$ correction to classical thermodynamics

In the last section we saw that at very high energy density the thermodynamics is degenerate — the temperature is equal to the Hagedorn one (2.11) independently of the energy density μ. It is thus of interest to compute subleading corrections to the equation of state. As discussed in the introduction, one expects the entropy-energy relation to take the form

$$S(E) = \beta_H E + \alpha \log \frac{E}{\Lambda} + O\left(\frac{1}{E}\right) , \tag{3.1}$$

where Λ is a dimensionful constant (a UV cutoff) which we will not keep track of below. Consider the canonical partition sum

$$Z(\beta) = \int_0^\infty dE \rho(E) e^{-\beta E} . \tag{3.2}$$

Near the Hagedorn temperature one might expect $Z(\beta)$ to be dominated by the contributions of high energy states;[3] if this is the case, one can replace $\rho(E)$ by (1.2) and find,

$$Z(\beta) \simeq \int dE E^\alpha e^{(\beta_H - \beta)E} \simeq (\beta - \beta_H)^{-\alpha-1} . \tag{3.3}$$

The free energy (2.13) is thus given by

$$\beta \mathcal{F} \simeq (\alpha + 1) \log(\beta - \beta_H) . \tag{3.4}$$

The energy computed in the canonical ensemble[4] is

$$E = \frac{\partial(\beta\mathcal{F})}{\partial\beta} \simeq \frac{\alpha+1}{\beta - \beta_H} ; \tag{3.5}$$

thus the free energy (3.4) can be written as

$$-\beta\mathcal{F} \simeq (\alpha + 1) \log E . \tag{3.6}$$

Comparing to the expansion (2.13)–(2.15) we see that the leading term in the free energy arises from the torus (one loop) diagram in the background (2.7), since it scales as μ^0, like Z_1 in (2.14). In this section we will compute this term and determine α.

The torus partition sum in the background (2.7) is in fact divergent, since it is proportional to the infinite volume of the cigar, associated with the region far from

[3] In section 4 we will see that this assumption is valid slightly *above* the Hagedorn temperature, but is *not* valid slightly below it.

[4] Comparing to (1.5) we see that the canonical and microcanonical ensembles are not equivalent. This is a familiar feature of systems with a Hagedorn density of states. We will return to it in section 4.

the tip, $\phi \to \infty$. As is standard in other closely related contexts, we will regulate this divergence by requiring that

$$\phi \leq \phi_{UV} \,. \tag{3.7}$$

In the fivebrane theory, this can be thought of as introducing a UV cutoff. This makes the partition sum finite, but the bulk of the amplitude still comes from the region far from the tip of the cigar. For the purpose of computing this "bulk contribution" one can replace the cigar by a long cylinder with ϕ bounded on one side by the UV cutoff (3.7) and on the other by the location of the tip of the cigar. Combining (1.1) and (2.15) we find that

$$\frac{1}{Q}\log\frac{\mu}{N} \leq \phi \leq \phi_{UV} \,. \tag{3.8}$$

Thus, the length of the cut-off cylinder is

$$L_\phi = \phi_{UV} - \frac{1}{Q}\log\frac{\mu}{N} = -\frac{1}{Q}\log E + \text{const} \,. \tag{3.9}$$

Since we are only interested in the energy dependence, we suppress in (3.9) a large energy independent contribution. Any contributions to the torus partition sum from the region near the tip of the cigar can also be lumped into this constant. Note the minus sign in front of $\log E$ in (3.9). The length L_ϕ is of course positive; the minus sign simply means that L_ϕ decreases as E grows.

To recapitulate, for the purpose of calculating the bulk contribution to the torus partition sum, we can replace the background (2.7) by

$$\mathbb{R}_\phi \times S^1 \times \mathrm{SU}(2)_N \times \mathbb{R}_5 \,. \tag{3.10}$$

The linear dilaton direction is regulated as in (3.8). The circumference of the S^1 is β_H (2.11).

The background (3.10) is easy to analyze since it is very similar to that describing flat space at finite temperature (see e.g. [16, 17, 18]). The bosonic fields on the worldsheet are seven free fields, one of which (euclidean time) is compact, and a level $N-2$ SU(2) WZW model. The worldsheet fermions are free and decoupled from the bosons; their partition sum, and in particular the sum over spin structures, is the same as in the flat space analysis, which we briefly review next.

Collecting all the contributions to the thermal torus partition sum in the background (3.10) we find,[5]

$$\begin{aligned} Z_1 = {} & \frac{\beta V_5 L_\phi}{4}\int_F \frac{d^2\tau}{\tau_2}\left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^{7/2}\frac{1}{|\eta(\tau)|^{10}} Z_{N-2}(\tau)\, \times \\ & \times \sum_{n,m\in\mathbb{Z}}\sum_{\mu,\nu=1}^{4}\delta_\mu U_\mu(n,m)\delta_\nu U_\nu(n,m)\left(\frac{\vartheta_\mu(0,\tau)}{\eta(\tau)}\right)^4\left(\frac{\vartheta_\nu(0,\bar\tau)}{\eta(\bar\tau)}\right)^4 e^{-S_\beta(n,m)} \,. \end{aligned} \tag{3.11}$$

[5]We follow the conventions of [18], which should be consulted for additional details. We also drop the subscript H on β_H, and will reinstate it later.

The modular integral runs over the standard fundamental domain F. Z_{N-2} is the partition sum of level $N-2$ SU(2) WZW [6] (see for example [19]),

$$Z_{N-2}(\tau) = \sum_{m=0}^{N-2} \chi_m^{(N-2)}(q)\chi_m^{(N-2)}(\bar{q}) = \sum_{m=0}^{N-2} |\chi_m^{(N-2)}(q)|^2 \,, \tag{3.12}$$

where $q = \exp(2\pi i\tau)$ and

$$\chi_m^{(N-2)}(q) = \frac{q^{(m+1)^2/4N}}{\eta(q)^3} \sum_{n\in\mathbb{Z}} [1+m+2nN)]q^{n(1+m+Nn)} \,. \tag{3.13}$$

We note for future reference that Z_{N-2} is real and positive.

μ, ν denote the spin structure for left and right moving worldsheet fermions, respectively. $\delta_\mu = (\pm, -, +, -)$ are signs coming from the usual GSO projections for IIA and IIB superstrings at zero temperature; n, m are winding numbers of euclidean time around the two non-contractible cycles of the torus. The soliton factor $S_\beta(n,m)$ is given by

$$S_\beta(n,m) = \frac{\beta^2}{4\pi\alpha'\tau_2}(m^2 + n^2|\tau|^2 - 2\tau_1 mn) \,. \tag{3.14}$$

$U_\mu(n,m)$ are additional signs that are associated with finite temperature. Their role is to implement the standard thermal boundary conditions, that spacetime bosons (fermions) are (anti-)periodic around the euclidean time direction. One can show [18] that this requirement together with modular invariance leads to:

$$\begin{aligned}
U_1(n,m) &= \frac{1}{2}\left(-1 + (-1)^n + (-1)^m + (-1)^{n+m}\right) \\
U_2(n,m) &= \frac{1}{2}\left(1 - (-1)^n + (-1)^m + (-1)^{n+m}\right) \\
U_3(n,m) &= \frac{1}{2}\left(1 + (-1)^n + (-1)^m - (-1)^{n+m}\right) \\
U_4(n,m) &= \frac{1}{2}\left(1 + (-1)^n - (-1)^m + (-1)^{n+m}\right) \,.
\end{aligned} \tag{3.15}$$

The terms with $\mu = 1$ in (3.11) vanish because of the presence of fermionic zero modes for the $(+,+)$ spin structure, or equivalently since $\vartheta_1(0,\tau) = 0$.

The torus partition sum (3.11) can be rewritten in a way that makes it manifest that the coefficient of $\beta V_5 L_\phi/4$ is positive,

$$\begin{aligned}
Z_1 = \frac{\beta V_5 L_\phi}{4} \int_F \frac{d^2\tau}{\tau_2} \left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^{7/2} \frac{1}{|\eta(\tau)|^{18}} Z_{N-2}(\tau) \times \\
\times \sum_{n,m\in\mathbb{Z}} \left|\sum_{\mu=2}^{4} U_\mu(n,m)\delta_\mu \vartheta_\mu^4(0,\tau)\right|^2 e^{-S_\beta(n,m)} \,.
\end{aligned} \tag{3.16}$$

[6] We choose the A series modular invariant; the D and E series modular invariants can also be studied and correspond to other vacua of LST [4].

It is not difficult to check that the integral (3.16) is convergent at $\tau_2 \to \infty$, the only region where a divergence could occur.

To exhibit the interpretation of (3.16) as a sum over the free energies of physical string modes one can proceed as follows [13, 16, 17]. Using the modular invariance of the integrand and the covariance of (n, m), one can extend the integral from the fundamental domain to the strip

$$S: \quad -\frac{1}{2} \leq \tau_1 \leq \frac{1}{2}; \qquad \tau_2 \geq 0\,, \tag{3.17}$$

while restricting to configurations with $n = 0$ in (3.16). This leads to

$$\begin{aligned} Z_1 = \frac{\beta V_5 L_\phi}{4} \int_S \frac{d^2\tau}{\tau_2} \left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^{7/2} \frac{1}{|\eta(\tau)|^{18}} Z_{N-2}(\tau) \times \\ \times \sum_{m=-\infty}^{\infty} \left| \sum_{\mu=2}^{4} U_\mu(0,m)\delta_\mu \vartheta_\mu^4(0,\tau) \right|^2 e^{-S_\beta(0,m)}\,. \end{aligned} \tag{3.18}$$

The integral over τ_1 projects on physical states (i.e. those with $L_0 = \bar{L}_0$), while τ_2 plays the role of a Schwinger parameter. Because of the Jacobi identity $\vartheta_2^4(0,\tau) - \vartheta_3^4(0,\tau) + \vartheta_4^4(0,\tau) = 0$, and the fact that $U_2(0,m) = (-)^m$, $U_3(0,m) = U_4(0,m) = 1$, the sum over m in (3.18) can be restricted to odd integers. It is not difficult to check in this representation too that the integral over τ_2 is convergent.

We are now ready to determine the parameter α in (3.1), (3.6). Using the relation (2.13) between the free energy $\mathcal{F}$ and the string partition sum, as well as (3.6), we see that Z_1 should be proportional to $\log E$. This is indeed the case in (3.18) since the length L_ϕ goes like $-\log E$ (see (3.9)). Combining these relations we find that

$$\begin{aligned} \alpha + 1 = -\frac{\beta V_5}{4Q} \int_S \frac{d^2\tau}{\tau_2} \left(\frac{1}{4\pi^2\alpha'\tau_2}\right)^{7/2} \frac{1}{|\eta(\tau)|^{18}} Z_{N-2}(\tau) \times \\ \times \sum_{m=-\infty}^{\infty} \left| \sum_{\mu=2}^{4} U_\mu(0,m)\delta_\mu \vartheta_\mu^4(0,\tau) \right|^2 e^{-S_\beta(0,m)}\,. \end{aligned} \tag{3.19}$$

We see that $\alpha + 1$ is negative, as stated above.[7] Physically, it is clear that it is counting the free energy of the perturbative string modes which live in the vicinity of the black brane. An interesting point which was mentioned in [6, 7] is that α is an extensive quantity — it is proportional to the volume of the fivebrane V_5, in contrast, say, to the one particle free energy in critical string theory, where the analogous quantity is of order one.

[7] Of course, since the r.h.s. of (3.19) is proportional to V_5 which is assumed to be very large, we can neglect the $+1$ on the left hand side.

The integral (3.19) appears in general to be rather formidable and we do not know whether it can be performed exactly. In the remainder of this section we will compute it in the limit $N \to \infty$, where the computation simplifies.

For large N the partition sum corresponding to the three-sphere, $Z_{N-2}(\tau)$, simplifies significantly. Indeed, for $N \gg 1$ (3.12) can be approximated as

$$Z_{N-2}(\tau) = \frac{1}{|\eta(q)|^6} \sum_{p=0}^{\infty} |q|^{(p+1)^2/2N} (p+1)^2 \,. \tag{3.20}$$

Returning to the evaluation of α, (3.19), we have

$$\begin{aligned} \alpha + 1 = & -\frac{\beta V_5}{4Q} \left(\frac{1}{4\pi^2\alpha'}\right)^{7/2} \int_S \frac{d^2\tau}{\tau_2^{9/2}} \left|\frac{1}{\eta(\tau)}\right|^{24} \times \\ & \times \sum_{m\in 2\mathbb{Z}+1} \sum_{p=0}^{\infty} e^{-((p+1)^2\tau_2)/2N} (p+1)^2 e^{-(\beta^2 m^2)/(4\pi\alpha'\tau_2)} \left|\vartheta_2^4 + \vartheta_3^4 - \vartheta_4^4\right|^2 (0,\tau) \,. \end{aligned} \tag{3.21}$$

At this point it is useful to recall that the inverse temperature β in (3.21) is in fact the Hagedorn temperature of LST, (2.11). In the large-N limit, $\beta_H \sim \sqrt{N}$ becomes large (or, equivalently, the Hagedorn temperature is small in string units) and the exponential term in (3.21) suppresses the amplitude, unless τ_2 is large as well (of order N). Therefore, the τ integral in (3.21) is dominated by the large-τ_2 region, which corresponds to the free energy of the supergravity modes. To compute the integral we recall the asymptotic forms of the ϑ and η functions at large τ_2 (see e.g. [20])

$$\begin{aligned} \vartheta_2(0,\,\tau) &= \sum_{n=-\infty}^{\infty} q^{1/2(n-1/2)^2} = 2q^{1/8}(1+q+\ldots) \\ \vartheta_3(0,\,\tau) &= \sum_{n=-\infty}^{\infty} q^{1/2n^2} = 1 + 2q^{1/2} + \ldots \\ \vartheta_4(0,\,\tau) &= \sum_{n=-\infty}^{\infty} (-1)^n q^{1/2n^2} = 1 - 2q^{1/2} + \ldots \\ \eta(\tau) &= q^{1/24} \prod_{n=1}^{\infty} (1-q^n) = q^{1/24} + \ldots \end{aligned} \tag{3.22}$$

Plugging in (3.21) and using the definition of the modified Bessel function

$$K_\nu(z) = \frac{1}{2}\left(\frac{2}{z}\right)^\nu \int_0^\infty t^{\nu-1} e^{-z^2/4t-t} dt \,, \tag{3.23}$$

we find

$$\begin{aligned} \alpha + 1 &= -\frac{8V_5}{\pi^6 (N\alpha')^{5/2}} \sum_{k,p=0}^{\infty} \left(\frac{2\pi(2k+1)^2}{(p+1)^2}\right)^{-7/4} (p+1)^2 K_{-7/2}(\sqrt{2\pi}(p+1)(2k+1)) \\ &\simeq -4.08 \cdot 10^{-4} V_5 (N\alpha')^{-5/2} \equiv -a_1 V_5 . \end{aligned} \tag{3.24}$$

Note that, as expected, a_1 is positive. Of course, as is clear from (3.21), we can write $\alpha + 1$ as $-a_1 V_5$ with a_1 a positive constant for all N, but in general a_1 receives contributions from massive string modes and is thus given by a complicated modular integral. The large-N behavior of a_1 is simpler and is given by (3.24).

The fact that α goes like $N^{-5/2}$ for large N was found in a different way in [6], by analyzing the deformation of the classical solution (2.5) at one string loop. Our analysis determines the coefficient of $N^{-5/2}$, and in particular its sign which, as mentioned above, is important for the thermodynamics.

In the discussion above, the fivebrane was assumed to be effectively non-compact. It is interesting to study the thermodynamics of fivebranes wrapped around compact manifolds, and in particular the dependence of α on the size and shape of the manifold. As an example of the sort of dependence one can expect, consider compactifying the fivebrane on $(S^1)^5$ where all five circles have the same radius R. It is sufficient to consider the case $R \geq \sqrt{\alpha'}$ since smaller radii give rise to the same physics due to T-duality.

As is standard in string theory, the effect of this is to replace the contribution of the non-compact zero modes on R^5 by the momentum and winding sum on $(S^1)^5$:

$$\frac{V_5}{(4\pi^2\alpha'\tau_2)^{5/2}} \longrightarrow \left(\sum_{l,p\in\mathbb{Z}} q^{\alpha'/4(l/R+pR/\alpha')^2} \bar{q}^{\alpha'/4(l/R-pR/\alpha')^2} \right)^5 . \tag{3.25}$$

Consider for simplicity the limit $N \to \infty$ discussed above. As mentioned after eq. (3.21), since the Hagedorn temperature is very low, the modular integral is dominated in this case by $\tau_2 \sim N$. If the radius R is much larger than $\sqrt{N\alpha'}$, the sum over momenta on the r.h.s. of (3.25) can be approximated by an integral and gives the same contribution as in the non-compact case (namely the l.h.s. of (3.25)). For $R \sim \sqrt{N\alpha'}$ one has to include a few low lying momentum modes — this is a transition region. For $\sqrt{\alpha'} < R \ll \sqrt{N\alpha'}$ one can neglect all contributions of momentum (and winding) modes just like one is neglecting the contributions of oscillator states. Thus, we get in this case

$$\begin{aligned}
\alpha + 1 &= -\frac{\beta}{2Q}\left(\frac{1}{4\pi^2\alpha'}\right)\int_0^\infty \frac{d\tau_2}{\tau_2^2} \cdot 1024 \sum_{k,p=0}^{\infty} e^{-(\beta^2(2k+1)^2)/(4\pi\alpha'\tau_2)-((p+1)^2\tau_2)/2N} \\
&= -\frac{256}{\pi}\sum_{k,p=0}^{\infty}\left(\frac{2\pi(2k+1)^2}{(p+1)^2}\right)^{-1/2}(p+1)^2 K_{-1}(\sqrt{2\pi}(p+1)(2k+1)) \\
&\simeq -3.693\,.
\end{aligned} \tag{3.26}$$

Interestingly, we find that for small fivebranes α is independent of the number of fivebranes N in the $N \to \infty$ limit. Note also that in this case it is important to keep the $+1$ on the l.h.s. of (3.26), since α is of order one.

To summarize, the power α that appears in the high energy density of states (1.4) exhibits an interesting dependence on the size of the spatial manifold that the fivebranes are wrapping. For manifolds of size much larger than the characteristic scale of LST, $\sqrt{N\alpha'}$, α is proportional to the volume of the manifold, while for sizes much smaller than this charateristic scale, it saturates at a finite value, which is independent of N (for large N), (3.26). If the density of states (1.4) is due to strings confined to the fivebranes, then these strings belong to a new universality class, with typical configurations not exceeding the size $\sqrt{N\alpha'}$. It would be interesting to understand this universality class better (see also [6]).

4. Comments on the near-Hagedorn thermodynamics of LST

The main result of our discussion so far is that the thermodynamics corresponding to non-extremal fivebranes is unstable. The temperature-energy relation has the form (1.5), with α given by (3.21) or for large N by (3.24), (3.26). Since it is negative, the temperature is above the Hagedorn temperature, and the specific heat is negative. This raises two immediate questions:

1. What is the thermodynamics for temperatures slightly below the Hagedorn temperature?

2. What is the nature of the instability above the Hagedorn temperature?

The purpose of this section is to discuss these issues. Consider first the behavior well below the Hagedorn temperature, $\beta \gg \beta_H$. In this regime, the thermodynamics is expected to reduce to that corresponding to the extreme IR limit of LST, which is the $(2,0)$ six dimensional SCFT for type IIA LST, or six dimensional $(1,1)$ SYM for IIB. From the point of view of the holographic description, this regime corresponds to the strong coupling region of the near-horizon geometry of the fivebranes [4], and thus should not be visible in the perturbative theory on the cigar (2.5).

What happens as the temperature approaches T_H from below? One might expect that due to the Hagedorn growth in the density of states (1.4), the high energy part of the spectrum dominates as $\beta \to \beta_H$, and the partition sum is well approximated by (3.3). What actually happens depends on the value of α, as we discuss next.

Consider first the case of large V_5 ($R \gg \sqrt{N\alpha'}$ in the discussion at the end of section 3). In this case, $|\alpha|$ is large, and the contribution to the partition sum of the high energy part of the spectrum, (3.3), goes rapidly to zero as $\beta \to \beta_H$. The integral over E is dominated by states with moderate energies, whose contribution to the partition sum is analytic at β_H. It is clear that the mean energy remains finite as we approach the Hagedorn temperature from below, and that thermodynamic fluctuations are suppressed (by a factor of the volume V_5). Since the Hagedorn temperature is reached at a finite energy, it corresponds to a phase transition.

As V_5 decreases, α decreases as well, until it reaches the value (3.26). The fluctuations in energy in the canonical ensemble increase with decreasing α. To see that, consider the case $R \ll \sqrt{N\alpha'}$ in the discussion at the end of section 3. Since $-5 < \alpha < -4$ in that case, the expectation values $\langle E^n \rangle$ with $n \geq 4$ in the canonical ensemble diverge as

$$\langle E^n \rangle \sim (\beta - \beta_H)^{-\alpha - n - 1}\,. \tag{4.1}$$

In such situations, one is instructed to pass to the microcanonical ensemble, in which the energy is fixed and the temperature is defined by (1.5). The perturbative evaluation of β in (1.5) gives a temperature *above* the Hagedorn temperature. This of course does not imply that LST cannot be defined at temperatures below T_H; instead, it means that to study the theory at such temperatures one must compute $S(E)$ to all orders in $1/E$, include non-perturbative corrections, and solve the equation

$$\beta = \frac{\partial S(E)}{\partial E}$$

to find the energy E corresponding to a particular $\beta > \beta_H$. From the form of the leading terms in $S(E)$ it is clear that the solution of this equation will correspond to finite E. We are led again to the conclusion that the Hagedorn temperature is reached at a finite energy and thus is associated with a phase transition.

Since the study of the non-extremal fivebrane geometry in the previous sections is perturbative in $1/E$, it is not useful for studying the regime $\beta > \beta_H$. Nevertheless, it seems clear that the specific heat is positive there (this is certainly the case for the infrared theory on the fivebranes). Furthermore, since the energy — temperature relation is such that the Hagedorn temperature is reached at a finite energy, we are led to the second question raised in the beginning of this section: what is the nature of the high temperature phase of LST?

The perturbative analysis of the near-extremal fivebrane, which is valid for β slightly below β_H, predicts that the thermodynamics is unstable. Usually, in such situations the instability is associated with a negative mode in the euclidean path integral (a tachyon). Examples include the instability of flat space at finite temperature in Einstein gravity [21], and the thermal tachyon that appears above the Hagedorn transition in critical string theory. The one loop instability found in section 3 leads one to believe that a similar negative mode should appear in LST above the Hagedorn temperature.

At first sight this statement appears surprising. For large but finite N, far from the tip of the cigar (2.5), the near-horizon geometry (2.7) is essentially the same as in critical string theory at the temperature T_H (2.11), which goes to zero in the limit $N \to \infty$. There are clearly no tachyons in critical string theory at low temperature; thus we conclude that any unstable modes of the thermal background (2.7) must be localized near the tip of the cigar, i.e. near $\sigma = 0$ in the coordinates (2.5). This is natural, since one expects a phase transition to change the structure of the horizon of the black brane; the asymptotic behavior far from the tip of the cigar (a cylinder with circumference β) should remain unchanged.

States localized near the tip of the cigar in LST were studied in [5]. A convenient way to study them is to construct observables which correspond to vertex operators whose wave-functions are non-normalizable at $\phi \to \infty$ (which can be thought of as off-shell observables in LST) and compute their correlation functions. Normalizable states on the cigar, which are created by these observables acting on the vacuum, give rise to poles in these correlation functions. The masses of these states can be read off the locations of the poles.

In our case, it turns out that the observable that creates the light state when acting on the vacuum is the "fermionic string tachyon," whose vertex operator in the $(-1,-1)$ picture is

$$\mathcal{T}_m(\vec{p}) = e^{-\varphi-\bar{\varphi}} V_{j;m,m} e^{i\vec{p}\cdot\vec{x}}\,. \tag{4.2}$$

The notation here is the following (see [5] for further details). φ, $\bar{\varphi}$ are the bosonized superconformal ghosts. $\vec{p}$ is the spatial momentum along the fivebrane. $V_{j;m,m}$ is an observable in the cigar CFT. It belongs to a large class of primaries in the cigar SCFT corresponding to momentum and winding modes $V_{j;m,\bar{m}}$, where $(m,\bar{m})$ are related to the momentum and winding numbers around the cigar, $n, w \in Z$, via

$$m = \frac{1}{2}(n + wN); \qquad \bar{m} = -\frac{1}{2}(n - wN)\,. \tag{4.3}$$

The worldsheet scaling dimensions of these observables are

$$\Delta_{j;m,\bar{m}} = \frac{m^2 - j(j+1)}{N}; \qquad \bar{\Delta}_{j;m,\bar{m}} = \frac{\bar{m}^2 - j(j+1)}{N}\,. \tag{4.4}$$

In particular, we see that the observable (4.2) corresponds to a pure winding mode around euclidean time, with winding number $w = 2m/N$.

The mass shell condition for the vertex operator (4.2) is

$$\frac{\alpha'}{4}|\vec{p}|^2 + \frac{m^2 - j(j+1)}{N} = \frac{1}{2}\,. \tag{4.5}$$

This equation can be thought of as determining j as a function of $|\vec{p}|$ and m. The operators $\mathcal{T}_m(\vec{p})$ thus correspond to off-shell observables in LST [5].

Not all observables (4.2) are physical. The GSO projection implicit in the partition sum (3.11) projects out those with even winding number w, leaving behind those with $w \in 2Z+1$. This is analogous to the situation in flat space where the finite temperature GSO projection projects out tachyons with even winding number. This analogy suggests that the operator (4.2) with winding number one creates from the vacuum the thermal tachyon for $\beta < \beta_H$. We will next show that this is indeed the case.

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As explained in [5], the two point function of the operator (4.2) has a simple pole whenever m and j belong to a principal discrete series representation[8] of SL(2), i.e. for

$$m = j + l; \qquad l = 1, 2, 3, \ldots \tag{4.6}$$

The lowest mass state corresponds to $l = 1$; plugging into (4.5) we see that the corresponding mass-shell condition is

$$\frac{\alpha'}{4}|\vec{p}|^2 + \frac{w-1}{2} = 0\,. \tag{4.7}$$

Thus, the mass is

$$\frac{\alpha'}{4}M_w^2 = \frac{w-1}{2}\,. \tag{4.8}$$

The mode with winding number zero would have corresponded to a tachyon had it existed, but it is projected out by GSO. The winding number one ($w = 1$) mode is massless; the higher (odd) winding number modes are massive. It is not difficult to show that the masses of all states obtained by repeating the above procedure for other observables are strictly positive.

Since the vacuum we are studying is non-supersymmetric, it is reasonable to expect that the masses (4.8) receive quantum corrections. In particular, it is likely that the classically massless state with $w = 1$ is lifted at one loop (see (2.14), (2.15)):

$$M_1^2 = Ce^{2\Phi_0} + O(e^{4\Phi_0}) = \frac{CN}{\mu} + O\Big(\frac{1}{\mu^2}\Big)\,. \tag{4.9}$$

We have not computed these corrections, but would like to argue that $C < 0$, so that string loop effects drive the massless state tachyonic. This would lead to a consistent picture of the high temperature phase of LST. The perturbative thermodynamics is marginally stable classically (1.2), and is destabilized at one loop (1.4). Correspondingly, the classical euclidean theory contains a zero mode winding once around euclidean time, and one loop effects turn it into a negative mode.

Needless to say, it would be interesting to verify the conjecture that C is negative by an explicit one loop calculation. Assuming that this is indeed the case, we arrive at the picture described earlier in the paper: the fivebranes reach the Hagedorn temperature at a finite energy density. At that point the mode with winding number one described above turns tachyonic and condenses. The system thus undergoes a phase transition.

The precise temperature at which this condensation occurs depends on the behavior of the tachyon potential for $\beta \simeq \beta_H$. The quadratic term in the potential was argued above to change sign at β_H. The physics of the phase transition depends on the sign of the quartic term. It would be interesting to compute this term directly. It would also be nice to describe the endpoint of tachyon condensation. Since our

[8] There are bounds on j which are discussed in [5]; they are satisfied here and will not be reviewed.

description of this mode is rather indirect (as a pole in a correlation function of the observables (4.2)), and the condensation involves understanding string loop effects, this remains an interesting challenge.

Acknowledgments

We thank O. Aharony for useful discussions and comments on the manuscript. This work was supported in part by DOE grant #DE-FG02-90ER40560.

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